CURRENT RESEARCH IN SOCIAL PSYCHOLOGY

A MODIFICATION OF PONTON'S EQUATION FOR CALCULATING PROBABILITIES IN NORMAL DISTRIBUTIONS

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ABSTRACT

Ponton published a formula for approximating the area under the normal curve represented by any given z. The current paper describes a change that increases the accuracy of these estimates, and a Windows program that uses the modified formula to calculate the area.

INTRODUCTION

Ponton (1998) provided a formula in this journal for estimating with a calculator the probability associated with any value of z. While his formula worked well, it provided results slightly different from published values for scores he identified as being of particular interest in the social and behavioral sciences. His formula for determining the area under the normal curve is:

Area = K * sqrt [1 – e^ (-(z^2) / 2)], (1)

where e approximately = 2.7182818…, and

K = 0.5 + [1 / sqrt (Pi) – 0.5] * [e^ (-(z^2) / sqrt (2 * Pi))]. (2)

Ponton compared results using equation 2 with published values (Beyer, 1981), where z resulted in probabilities of approximately .05, .01, .005 and .001. He also provided a simplified formula for computing K:

K = 0.5 + 0.064 * [e^ (-0.4 * (z^2))]. (3)

He suggested increasing the accuracy of equation 3 by replacing 0.064 with 0.066 for certain values of z. I was interested in further increasing the accuracy of these estimates for the z-scores presented by Ponton and wrote a Windows program using his equations.

RESULTS

Using my program with Ponton's equations I obtained p values identical to those he published. I found that the accuracy of equation 3 could be improved by replacing 0.064 with x:

K = 0.5 + x * [e^ (-0.4 * (z^2))] (4)

where the value of x depends on the value of z such that:

If Z	Then X =
<2.0	0.0665
>=2.0 and <2.5	0.063
>=2.5 and <3.0	0.06
>=3.0	0.05

Equation 4 performed better than equation 3, achieving results identical to Ponton's published criteria from Beyer (1981):

Z	Equation 3	Beyer	Equation 4
1.65	0.0501	0.0495	0.0495
2.33	0.0097	0.0099	0.0099
2.57	0.0048	0.0051	0.0051
3.10	0.0007	0.0010	0.0010

Although the values of x were maximized for these four values of z, they were less accurate for other values, particularly for those representing the tails of the normal distribution. I then identified eight values of x such that:

If Z	Then X =
<=0.75	0.065
>0.75 and <1.0	0.07
>=1.0 and <1.25	0.0667
>=1.25 and <2.0	0.0665
>=2.0 and <2.5	0.063
>=2.5 and <3.0	0.06
>=3.0 and <3.25	0.05
>=3.25	0.045

By selecting from these eight values of x, equation 4 produced estimates accurate to a minimum of two places when compared to published results (Meredith, 1967) for a wide range of values of z, while still agreeing to four places for the values of z presented by Ponton:

Z	Meredith	Equation 4
0.00	.5000	.5000
0.25	.4013	.4012
0.50	.3085	.3084
0.75	.2266	.2265
1.00	.1587	.1583
1.25	.1056	.1056
1.50	.0668	.0669
1.75	.0401	.0401
2.00	.0228	.0232
2.25	.0122	.0123
2.50	.0062	.0063
2.75	.0030	.0029
3.00	.0013	.0014
3.25	.0006	.0006

DISCUSSION

Ponton (1998) developed equations to estimate probabilities with a calculator, which provided a good approximation to published values. I found that a small change in his equation increased the accuracy of these estimates still more for the values he identified as of special interest to psychologists, as well as for a large range of z-scores. The number of values of x could be increased still more to further improve the accuracy of equation 4, but with a corresponding decrease in simplicity of use. In my clinic we use equation 4 to compute a confidence level for test score changes during therapy. Two decimal places of accuracy are sufficient when the object is to assign confidence levels, e.g., 57%, 81% or 98%, but might not be sufficient under other circumstances.

The Probability Estimator program used for the above estimations has a minimal user interface in the interest of simplicity and size. The user enters the z-score in the first text box, clicks a command button or presses the Enter key, and a second text box displays the estimated 1-tailed probability for that z-score. The program uses all eight values of x listed above. The Estimator offers two advantages over using a calculator. First, it is easier and faster to use. Second, because it runs in a small window, the Estimator can remain visible while the psychologist is working in another window that occupies less than the full screen. This facilitates copying z from the second window, pasting it into the Estimator, copying the estimated p from the Estimator and pasting p into the second window. The executable program requires about 20KB of drive space. Those who prefer to use the Estimator rather than a calculator for obtaining these estimates may contact me at the address below and I will attach the necessary files to my reply.

REFERENCES

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Meredith, W. H. 1967. *Basic Mathematical and Statistical Tables for Psychology and Education.* New York: McGraw-Hill.

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